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Volume 4 | No. 1 | 2018

SALU-Commerce &  
Economics Review  
[www.cer.salu.edu.pk](http://www.cer.salu.edu.pk)

**The Random Walk Hypothesis for BRICS and Pakistan: New Evidence  
Based on Variance Ratio Tests**

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***Abstract:-***

*This study attempts to re-examine the random walk hypothesis for BRICS-P countries; Brazil, Russia, India, China, South Africa and Pakistan by using daily stock returns ranging from January 2000 to March 2017. The hypothesis is tested through Variance Ratio Tests including the conventional Lo- MacKinlay, Chow Denning, new Wright's rank, Sign tests, Hang and Kim sub sampling tests. Results under all individual and joint testing methods show that the stock prices in sample countries do not follow the random walk. These findings indicate intertemporal predictability that relates with investors' astute. This study recommends investors to focus more to capture risk-adjusted abnormal returns and to devise their trading strategies accordingly.*

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**Keywords:** BRICS, Weak-form market efficiency, Random walk hypothesis,  
Emerging markets

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## 1. Introduction

Market efficiency has remained an extensively tested area for about a half century; when the price doesn't change due to any revealing information the market would be called efficient (Malkiel & Fama, 1970). Literature holds three different efficiencies; allocation efficiency, operational efficiency and informational efficiency. The later one is being addressed in this study on Brazil, Russia, India, China, South Africa (BRICS) and Pakistan. The sample countries are recognised as emerging and developing economies, newly industrialised and politically powerful at the global level. BRICS countries are the members of G-20. For the crucial role in world's economy, BRICS countries were very first coined by Sachs (2001) later on in a following paper Sachs (2003) attempted to figure out the future of world over the period of next 50 years in that same paper it is predicted that in the next 40 years the contribution of BRICS in world economy will be higher than the group of six current prominent countries (USA, Italy, Japan, UK, France and Germany). In this study, we add Pakistan along with BRICS, it holds two-folded reason, first in line with sample countries Pakistan is also emerging market and is regarded as one of the best stock markets of the world. Second, the principal member of BRICS, China has recently started C-PEC: China Pakistan Economic Corridor in Pakistan that can mutually push economies of both countries up.

This study attempts to find out the market efficiency in indices of mega players - BRICS-P, an area of keen interest for many researchers globally. Many researchers have been attracted to develop and test stock pricing behaviours' models. Those behaviours were notified as a fair game pattern at the start of last century that led to random walk hypothesis which states stock prices are unpredictable (Bachelier, 1900). The hypothesis was explored and studied by a couple of the researchers up to 1930's, further in 1960's that remained intensively debated topic. Currently, it is being explained by the Efficient Market Hypothesis (Fama, 1965) market efficiency is further classified in three levels; a weak form of efficiency, Semi-Strong efficiency and Strong Form of market efficiency based on the stock prices reflection of market information (Malkiel & Fama, 1970).

The random walk hypothesis is often linked with a weak form of market efficiency, where future changes in prices are independent of past price changes (Nalın & Güler, 2015). Besides, if markets are efficient by holding the random walk hypothesis, then stock prices will be a fair representation of intrinsic value or fundamental value estimates (Fama, 1995). To capture the random walk behaviour, various methodologies have been used around the globe. Wu (1996), and Laurence, Cai, and Qian (1997) employed serial correlation to come across with EMH, while Groenewold, Tang, and Wu (2003) applied unit root test method.

However, both of the methodologies were stated less powerful than earlier method of the Variance Ratio [VR] tests by Lo & Mac Kinlay, (1989). However, multiple VR tests were preferred over individual VR test. The random walk hypothesis is generally tested for several holding periods (k), where null hypothesis rejection is due to the rejection of some k values. Moreover, Chow and Denning (1993) emphasise the avoidance of size distortion problem via the employment of

sequential procedure of multiple Variance ratio tests. Still, a significant issue of asymptotic approximations for finite sample lead towards biased and misleading statistical inference with individual or multiple variance ratio tests linger. In last decade the studies of Whang and Kim (2003), three years late study of Kim (2006) recommended two alternatives; one the sub sampling technique while other addresses wild bootstrap of Chow Denning test as those does not dependent on asymptotic approximations.

This study contributes to the literature by re-investigating market efficiency via variance ratio test on BRICS countries and Pakistani stock market data.

Rest of the study is classified in a following way. Section II reviews the literature and covers previous research work; Section III goes over the data description and variable measurements. Section IV reports the methodology along with empirical results. Section V finally, draws the concluding remarks.

## **2. Literature Review**

Financial literature holds a substantial number of studies regarding random walk hypothesis and market efficiency, where emerging countries and developed countries remain a focus for researchers around the globe. The 19th century and early 2000's studies of Fama (1965), Samuelson (1965), Fama (1970), Black (1971), Summers (1986), Borges (2010) were conducted to find out the market efficiency at weak or semi-strong level, but the results were unconvincing. While, Appiah-Kusi and Menyah (2003) and Hall, Urga, and Zalewska-Mitura (1998) find both weak and semi-strong levels of efficiency in their studies. On the other hand, Lo and Mac Kinlay (1988) and Jarrett and Kyper (2006) found the inefficient stock market in their respective studies.

### ***Market Efficiency in Developing Markets***

A couple of methodologies were employed by several researchers in the pursuit of market efficiency across the globe, where Butler and Malaikah (1992) found Saudi' stock market inefficient while Kuwait stock market found efficient by using the autocorrelation method. Russian markets firstly evaluated inefficient and then efficient with the gap of two years by GARCH model to capture the time-varying trends by Hall *et al.*, (1998). An excellent study of Appiah-Kusi & Menyah, (2003) took weekly data of 11 African countries stock indices using E-GARCH model and come with a conclusion where 5 out of 11 African indexes hold the weak form of market efficiency and rest were reported inefficient in the study.

Further, Cooray and Wickremasinghe (2007) explored the weak form of efficiency for four Asian countries including; Pakistan, India, Sri Lanka and Bangladesh through unit roots tests; the study confirmed weak form of efficiency for all three countries other than Bangladesh. Furthermore as per Lim, Habibullah, and Hinich (2009) Chinese market experience a weak form of efficiency.

### ***Market Efficiency in Developed Markets***

Developing countries majorly put up with the weak form of market efficiency the study of Narayan and Smyth (2004) conducted through unit root test on the South Korean stock market is one of those shreds of evidence. During same year Worthington and Higgs (2004) did a thorough study of twenty European countries by utilizing multiple testing methods consisting of runs test, unit root ADF test, serial correlation and variance ratio test, results demonstrate five countries Sweden, Ireland, UK, Portugal and Germany support random walk in their respective indices, while the markets of Spain, Norway, Netherland along with France and Finland meet the random walk requirements up to slight level moreover, rest of the countries did not hold random walk in their stock markets.

More Borges (2010) examined market efficiency by means of serial correlation test, unit root test and variance ratio tests on five European countries embracing Germany, United Kingdom, France, Spain, Greece and Portugal, by using monthly and daily data, where no market efficiency found in Portugal, United Kingdom, Greece and France. While Germany and Spain stand with the weak form of efficiency.

### ***Market Efficiency in BRIC Markets***

Minimal number of studies has been conducted on BRIC countries yet, Chong, Cheng, and Wong (2010) inspected BRIC indices by means of diverse tests including Simple Moving Average, Momentum, Relative Strength Index and Moving Average Convergence Divergence, where Brazilian market came across as most efficient index than rest of the BRIC stock markets.

Further Mobarek and Fiorante (2014) examined BRIC markets by daily stock prices using bias-free statistical technique covering 15 years sample period inclusive of subprime crises and came up with the conclusion that these markets are fairly weak form efficient.

A detailed study of Hoque (2007) employed all VR tests where the sample consists of emerging countries other than BRICS-P. Another study of Nalın and Güler (2015) explored BRIC countries and Turkey by using Unit Root Analysis, Granger Causality test and monthly data. The results indicate the weak form of efficiency.

### ***The need for re-structuring the phenomenon***

Both random walk hypothesis and BRIC countries explored by number of studies under number of methodologies, however, none of the previous studies has incorporated all conventional individual Lo and MacKinlay (1989) Test, Test, Wright (2000) and multiple VR- Chow And Denning (1993) Test, Whang And Kim (2003) Test and Kim (2006) Test) on BRICS- P sample. The power transformed statistics inclusive sub sampling and bootstrapping methods and own sampling rank and sign tests have been evolved and modified over nineteen years' (1989- 2006) further this paper reviews the recent developments for testing random walk and martingale hypothesis through variance ratio tests. Here we present a platform for the restructuring of recent VR tests advancements on improving data sets of BRICS-P countries.

### 3. Data and Methodology

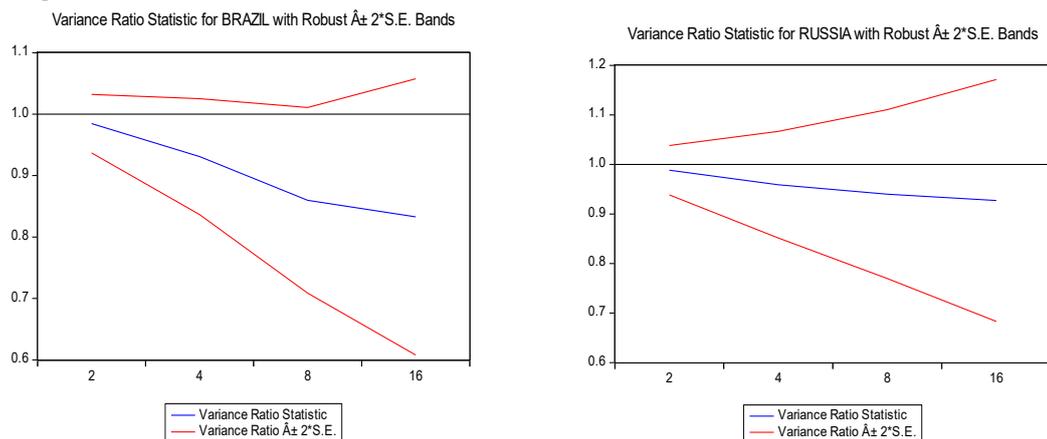
In search of market efficiency for BRICS countries and Pakistan, we get data of six countries stock exchanges; Boverspa Insex, RTS Index, BSE (Sensex) 30 Sensitive, Shenzhen SE Composite SUB, FTSE/JSE index and PSX 100 index respectively for Brazil, Russia, India, China, South Africa and Pakistan Indices. Data has been obtained from Bloomberg database, covering the period of January 2000 to March 2017. The criterion for choosing that sample is based on the availability of data for all indices taken into the sample.

We calculate stock returns by taking the first difference in the log of stock returns  $R_t = (\log(\frac{P_t}{P_{t-1}}))$  where  $P_t$  denotes the current stock price and  $P_{t-1}$  represents the last price in the index at time  $t$ , and  $R_t$  stands for the first difference of the log price. The log difference is calculated by following the methodology of (Chakravarty, Sarkar, & Wu, 1998).

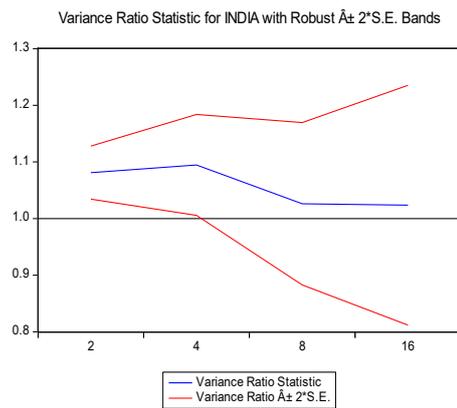
**Table 1 Stock Market Indices of BRICS- P and Their Sample Periods**

Countries	Index	Founded	From	To
Brazil	Brazil Boverspa	1895	1/1/ 2000	30/3/2017
India	BSE (Sensex) 30 Sensitive	1875	1/1/ 2000	30/3/2017
Russia	RTS Index	1995	1/1/ 2000	30/3/2017
China	Shenzhen SE Composite SUB	1990	1/1/ 2000	30/3/2017
South Africa	FTSE/JSE Africa	1887	1/1/ 2000	30/3/2017
Pakistan	PSX 100	1991	1/1/ 2000	30/3/2017

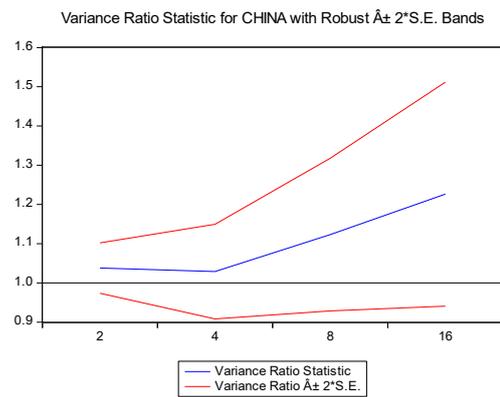
**Figure 1 to 6**



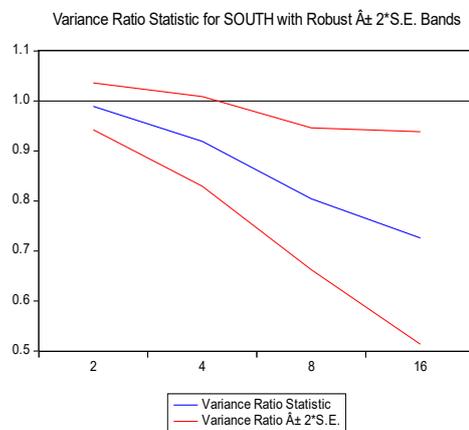
**Fig1. Brazil**



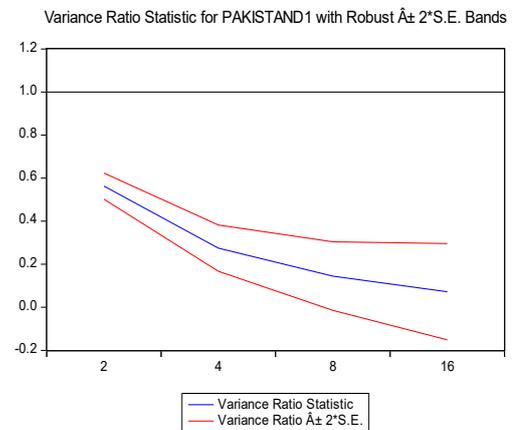
**Fig2. Russia**



**Fig3. India**



**Fig4. China**



**Fig5. South Africa**

**Fig 6. Pakistan**

### 3.1 The Variance Ratios Tests

Market efficiency is widely being tested via variance ratio test after the robust contribution of Lo & MacKinlay, (1989). We are following the methodology of Charles and Darné (2009) in this study. While testing The Random walk hypothesis under the variance ratio test, the variance is assumed to be linear in all sampling intervals (Charles & Darné, 2009) for example the sample variance of n-period return is n- times the sample variance of one period return. Further, the variance ratio at n lag is the ratio between (1/n) the of n- period return to the variance of the one period return therefore the variance ratio for the random walk should be equal to 1 for all of the values of n.

$$VR(x;n) = \frac{\left\{ (Tn)^{-1} \sum_{t=n}^T (x_t + \dots + x_{t-n+1} - n\hat{\mu})^2 \right\}}{\left\{ T^{-1} \sum_{t=1}^T (x_t - \hat{\mu})^2 \right\}} \quad (1)$$

Here  $x_t$  is the stock return at time  $t = 1 \dots T$  further  $x_t$  is considered as an insight of stochastic process following a martingale difference sequence. Where  $x_t$ 's are serially uncorrelated except the conditional or unconditional heteroscedasticity is being allowed there.

As per methodology of Wright (2000) the statistics of variance ratio will be written like  $t=1 \dots x_t$ . Variance ratio represented as  $V(n)$  that estimates an unknown population, it is the ratio of  $1/n$  times  $n$ - period return variance to the variance of 1- period return. So the value of  $VR(x;n)$  must be equal to the summation of all horizons of  $n$  if the stock returns are following random walk otherwise if the summation is less than one or greater than one of all horizon that indicate a negative and positive serial correlation respectively. More the negative serial correlation denotes the mean- reversion while the mean- aversion is the result of positive serial correlation.

### 3.2 Lo and MacKinlay' (1989) Test

Lo and MacKinlay (1988) are referred as individual variance ratio tests. Which recommend an asymptotic distribution of  $VR(x; k)$ , where under the assumption of conditional heteroscedasticity the  $n$  is being considered fixed and  $T$  equals to  $\infty$ . Additionally, the null hypothesis under the conditional heteroscedasticity demands  $V(k) = 1$ , and the test statistics  $M(x; k)$  is as below.

$$M(x; k) = \frac{VR(x;k)-1}{\phi^*(k)^{1/2}} \quad (2)$$

Further where,

$$\phi^*(k) = \sum_{t=j+1}^T \left[ \frac{2^{(k-j)}}{k} \right]^2 \delta(j)$$

$$\delta(j) = \left\{ \sum_{t=j+1}^T (x_t - \hat{u})^2 (x_{t-j} - \hat{u})^2 \right\} \div \left\{ \left[ \sum_{t=1}^T ((x_t - \hat{u})^2) \right]^2 \right\}$$

### 3.3 Chow and Denning (1993) Test

Due to Lo and MacKinlay (1988) test limitation of testing individual variance ratios at aggregate level for different values, which lacks the basic property of random walk hypothesis the ability to simultaneously check the variance ratio of all observation is equal to 1, Chow and Denning (1993) was preferred, where we test heteroscedasticity- robust null hypothesis for multiple variance ratio test;  $V(k_i) = 1$  for  $i=1 \dots$  alternative hypothesis states  $V(k_i) \neq 1$  for some holding period  $n_i$ .

$$MV(x; k_i) = \sqrt{T} \max_{1 \leq i \leq m} |M(x; k_i)| \quad (3)$$

Here the highest absolute value of individual Variance Ratio statistics determines the decision regarding null hypothesis. The distribution followed by the statistics is Studentized maximum modulus [SMM]; here the degree of freedom shown with  $m$  and  $T$ , further  $m$  denotes the number of  $n$  values. So we reject the null hypothesis when the value of  $T$  is large and if  $MV(x; n_i)$  is greater than  $[1 - (\alpha^*/2)]$ th percentile of the standard normal distribution where  $\alpha^* = 1 - (1 - \alpha)^{1/m}$

### 3.4 Wright (2000) Test

Lo-MacKinlay tests observed biased and right-skewed along with limited sampling distribution for the finite sample, due to this asymptotic limitation Wright (2000) proposed a variance ratio test based on signs and ranks with a nonparametric property to conventional asymptotic for variance ratio tests. For small size sample Wright's (2000) tests have two folded advantages over the Lo-MacKinlay test (i) No approximation need for asymptotic distribution due to exact sampling distribution of rank (R1 and R2) and sign (S1 and S2) tests (ii) the test is more robust in the comparison of traditional variance ratio tests. Under the i.i.d assumption the ranks based test does an exact job, show less size distortion, while the signs test goes exactly yet under conditional heteroscedasticity. The first difference variable is taken with  $T$  observations  $\{x_1, \dots, x_T\}$  further letting  $r(x)$  be the rank of  $x_T$  among  $(x_1, \dots, x_T)$ . Here the  $x_t$  is created from i.i.d sequence under the account of null hypothesis while with equal probability  $r(x)$  is a random permutation of  $1, \dots, T$  numbers.

The R1 and R2 as per Wright (2000) defined below.

$$R_1(k) = \left( \frac{(Tk)^{-1} \sum_{t=k}^T (r_{1,t} + \dots + r_{1,t-k+1})^2}{T^{-1} \sum_{t=k}^T r_{1,t}^2} - 1 \right) \times \phi(k)^{-1/2} \quad (4)$$

$$R_2(k) = \left( \frac{(Tk)^{-1} \sum_{t=k}^T (r_{2,t} + \dots + r_{2,t-k+1})^2}{T^{-1} \sum_{t=k}^T r_{2,t}^2} - 1 \right) \times \phi(k)^{-1/2} \quad (5)$$

Here the ranks  $r_{1,t}, r_{2,t}$  are

$$r_{1,t} = \frac{r(x_t) - \frac{T+1}{2}}{\sqrt{(T-1)(T+1)/12}}$$

$$r_{2,t} = \Phi^{-1} \frac{r(x)}{T+1}$$

Here  $\Phi^{-1}$  is the inverse form of standard normal cumulative distribution function Further R1 and R2 follow the precisely same distribution, so by simulating their exact distribution, the critical test values can be found.

The first difference signs test is

$$S_1(k) = \left( \frac{(Tk)^{-1} \sum_{t=k}^T (s_t + \dots + s_{t-k+1})^2}{T^{-1} \sum_{t=k}^T s_t^2} - 1 \right) \times \emptyset(k)^{-1/2} \quad (6)$$

$$S_2(k) = \left( \frac{(Tk)^{-1} \sum_{t=k}^T (s_t(\bar{\mu}) + \dots + s_{t-k+1}(\bar{\mu}))^2}{T^{-1} \sum_{t=k}^T s_t(\bar{\mu})^2} - 1 \right) \times \emptyset(k)^{-1/2} \quad (7)$$

Here  $s_t = 2u(x_t, 0)$ ,  $s_t(\bar{\mu}) = 2u(x_t, \bar{\mu})$

$$\text{And } u(x_t, q) = \begin{cases} 0.5 & \text{if } x_t > q \\ -0.5 & \text{otherwise} \end{cases}$$

The critical values of S1 and S2 test can be found out via simulating the exact sampling distribution likewise the R1 and R2 tests.

### 3.5 Whang and Kim (2003) Test

Another method to check random walk hypothesis through variance ratio test using the sub-sampling techniques was developed by (Whang and Kim, 2003). This method is preferred over conventional variance ratio tests due to its excellent properties (no or little distortion) with the relatively small sample.

The joint null hypothesis is being tested by following statistics.

$$MV_T = \sqrt{T} gN(x_1, \dots, x_T)$$

Where

$$g_t(x_1, \dots, x_T) = \max_{1 \leq i \leq m} |M_r(K_i)|$$

Here  $M_r(k_i) = VR(x; k)$  and  $VR(x; k)$  is as equation (1) further for  $MV_T$  Statistics the sampling distribution is written as follows:

$$G_T(x) = P(\sqrt{T} g_t(x_1, \dots, x_T) \leq x)$$

The complicated thing to estimate over here is the maximum of a multivariate normal vector with an unknown covariance matrix. Hence, previous researches of Charles (2009) propose the null hypothesis approximation via subsampling approach.

If the  $b$  is the size of subsample and  $t$  is time then  $t = 1, \dots, T-b+1$  and subsample is  $(x_t, \dots, x_{t-b+1})$  and the subsample calculation are shown as  $g_{T,b,t} = g_b(x_t, \dots, x_{t-b+1})$ . Further from  $g_{T,b,t}$ 's collection of all individuals subsamples following equation is estimated.

$$\hat{G}_{T,b}(x) = (T - b + 2)^{-1} \sum_{t=0}^{T-b+1} l(\sqrt{b}gT_{b,t} \leq x)$$

Here the  $l(\cdot)$  is an indicator function, so if its value is 1 it will be regarded satisfactory otherwise unsatisfactory when value inside the bracket is zero. Here the critical value calculation  $100(1 - \alpha)\%$  can be done like  $(1 - \alpha)$ th percentile of  $\hat{G}_{T,b}$ , further, the p-value of the test is estimated through  $1 - \hat{G}_{T,b}(MV_T)$ . If the p value is lesser than  $\alpha$  or  $MV_T$  is greater than critical value then null hypothesis  $V(k_i) = 1 (i = 1, \dots, m)$  would be rejected. Moreover, a choice of block length is being made to implement the sub-sampling technique. Whang and Kim (2003) recommended ‘an equally spaced grid with  $[2.5T^{0.3}, 3.5T^{0.6}]$  intervals generates a number of block lengths, where they find a choice of block length irrelevant with size and power properties.

### 3.6 Kim (2006) Test

A method wild bootstrap bit similar to the sampling distribution of Variance ratio statistics introduced by (Kim, 2006) which is applicable for the datasets with conditional and unconditional heteroscedasticity.

The wild bootstrap is applied to chow- Denning test  $MV(x; k_i)$  Variance ratio test. The test is conducted through three steps as follow.

(i) Form a sample for bootstrap with T observations  $x_t^* = \eta_t x_t (t = 1, \dots, T)$  where  $\eta_t$  denotes a random sequence with  $E(\eta) = 0$  and  $E(\eta^2) = 1$ .

(ii) From the bootstrap sample statistics  $MV(x^*; k_i)$  generated in step (i) calculate  $MV^* = MV(x^*; k_i)$

(iii) To form  $\{MV(j)\}_{j=1}^m$  a bootstrap test statistics distribution step (i) and (ii) will be repeated for m times.

For approximating  $MV(x; k_i)$  statistics sampling distribution the bootstrap distribution  $\{MV(j)\}_{j=1}^m$  is being used to form the original data when bootstrap distribution  $\{MV(j)\}_{j=1}^m$  is greater than  $MV(x; k_i)$  statistics, p-value test can be obtained. More to implement the wild bootstrap test we follow Kim (2006)  $\eta_t$  standard normal distribution.

## 4 Results and Discussion

### 4.1 Descriptive Statistics

The summary statistics for BRICS- P indices are reported in Table 1 calculation are made by taking the log of stock returns. The data of all countries are leptokurtic as it is considered a standard feature for daily stock returns; all countries in the sample are negatively skewed. For a sound check of financial time series data, the Lagrange Multiplier test on the residuals of ARMA model is being applied to check the autoregressive conditional heteroscedasticity (ARCH) based on Akaike and Schwarz information criterion lag length. Likewise, the most of the financial time series data the magnitude of residuals supports the existence of significant ARCH (10) by showing strong conditional heteroscedasticity in data.

**Table 2 Descriptive Statistics of Stock Returns**

	<b>Brazil</b>	<b>Russia</b>	<b>India</b>	<b>China</b>	<b>S-Africa</b>	<b>Pakistan</b>
Mean	0.000303	0.000590	0.000428	0.000183	0.000425	0.000850
Median	0.000692	0.000918	0.000977	0.000683	0.000686	0.001055
Std. Dev.	0.018186	0.021149	0.015111	0.016103	0.012285	0.013403
Skewness	-0.078810	-0.215526	-0.183076	-0.348291	-0.176180	-0.258285
Kurtosis	6.659384	17.63107	10.31371	7.732748	6.447524	6.980069
Jarque-Bera	2371.266	37869.40	9478.111	4044.769	2122.693	2847.053
ARCH(10)	122.41***	85.25***	60.60***	43.12***	108.21***	95.13***
Observations	4242	4242	4242	4242	4242	4242

### **Empirical Findings**

Widely used variance ratio tests brought following results when exploring the random walk hypothesis, by incorporating individual variance ratio the six test statistics of Lo and MacKinlay under homoscedasticity and heteroskedasticity correspondingly M1 and M2. The Wright's rank and rank score based tests R1 and R2 and finally sign test with and without bootstrap method S1 and S2 are used for the null hypothesis of Random walk hypothesis or (martingale). The null is rejected when more than two results are significant at any level (1%, 5% and 10%),

As per Wright (2000) findings, the power of R1 and R2 test is never less than S1 and S2, and Table 3 reports the strong rejection of null hypothesis at 1% significance for K 2, 4, 8 and 16. The rejection of random walk hypothesis is quite apparent for BRICS countries and Pakistan with very strong values at all K levels, it should be noted that values of test statistics (M1, M2, R1, R2, S1 and S2) are very strong at K 2 and gradually declining with K level 4, 8 and 16 for example test statistics for Brazil have values -31.75, -26.10, -19.36 and -13.98 at K 2, 4, 8 and 16 respectively further Table 2 reports all the values of test statistics carry negative sign. The holding period is, as employed by Hoque (2007), hence (2, 4, 8, 16) are considered short for testing mean reversion.

**TABLE 3 CRITICAL VALUES FOR WRIGHT'S: M1, M2, R1, R2, S1 AND S2**

	K	M1	M2	R1	R2	S1	S2
<b>Brazil</b>	2	-31.75***	-18.39***	-31.10***	-32.26***	-22.91***	-18.29***
	4	-26.10***	-15.89***	-25.27***	-26.29***	-18.20***	-15.81***
	8	-19.36***	-12.56***	-18.71***	-19.39***	-13.48***	-12.47***
	16	-13.98***	-9.37***	-13.57***	-14.00***	-9.67***	-9.29***
<b>Russia</b>	2	-30.70***	-11.32***	-29.07***	-30.32***	-20.57***	-11.29***
	4	-26.15***	-10.22***	-24.73***	-25.97***	-17.36***	-10.19***
	8	-19.25***	-8.21***	-18.21***	-19.11***	-12.94***	-8.19***
	16	-13.93***	-6.56***	-13.00***	-13.71***	-9.19***	-6.54***
<b>India</b>	2	-28.91***	-14.69***	-27.01***	-28.85***	-18.34***	-14.62***
	4	-25.46***	-13.77***	-23.78***	-25.20***	-16.80***	-13.71***
	8	-19.17***	-11.12***	-17.83***	-18.89***	-12.49***	-11.07***
	16	-13.82***	-8.54***	-12.95***	-13.58***	-9.30***	-8.50***
<b>China</b>	2	-31.16***	-17.08***	-29.19***	-30.71***	-20.60***	-17.08***
	4	-26.54***	-15.66***	-24.46***	-25.92***	-17.43***	-15.66***
	8	-19.32***	-12.34***	-17.98***	-18.89***	-12.73***	-12.34***
	16	-13.92***	-9.46***	-12.95***	-13.58***	-9.03***	-9.46***
<b>South Africa</b>	2	-30.77***	-17.92***	-29.78***	-31.16***	-20.94***	-17.70***
	4	-25.57***	-15.75***	-24.89***	-25.91***	-17.35***	-15.59***
	8	-19.22***	-12.35***	-18.53***	-19.35***	-13.07***	-12.23***
	16	-13.85***	-9.20***	-13.35***	-13.92***	-9.68***	-9.11***
<b>Pakistan</b>	2	-30.11***	-16.18***	-27.56***	-29.75***	-17.75***	-15.89***
	4	-25.44***	-14.55***	-23.21***	-24.92***	-15.06***	-14.29***
	8	-19.02***	-11.83***	-16.97***	-18.37***	-11.19***	-11.61***
	16	-13.85***	-9.23***	-12.17***	-13.26***	-7.69***	-9.08***

\* Statistical significance at the 10% level. \*\* Statistical significance at the 5% level. \*\*\* Statistical significance at the 1% level

Table 4 shows the findings of multiple joint VR tests including MV: Chow-Denning test; MV\*: Kim test (2006); WK: Whang –Kim test following Whang and Kim (2003) Further for wild bootstrap test (MV\*) Kim (2006) the number 1000 is meant for m the bootstrap replication. The block length for Whang and Kim (2003) sub sampling test is as per their recommended interval and spaced grid  $[2.5T^{0.3}, 3.5T^{0.6}]$ . Results for MV: Chow- Denning test represents a robust positive number for the whole sample of BRICS- P countries where Brazil holds strong evidence regarding the rejection of null random walk hypothesis with a value of 18.30, while Russia holds relatively weak but significant proof with a value of 11.30. Rest of the countries values ranges in between, more all the MV values are non-negative.

The Kim (2006) MV\* test results are in line with Chow- Denning test but with more robust and positive values for the whole sample here Brazil again holds highest significant values of 30.77 but here Pakistan undergoes the lowest result with 27.31 value, and all other sample countries are in between. The results for Whang and Kim sub sampling test are estimated on equal spaced intervals of  $[2.5T^{0.3}, 3.5T^{0.6}]$  resultantly the block length values we get in a study by the whole sample are (83, 113, 196, 226, 422, 452, and 874) for all BRICS-P countries. Results show all the pieces of evidence with every block length bears negative and significant value. Besides, point to be highlighted here is the WK values are decreasing as the block length increases.

**Table 4 Multiple VR Test Results**

	MV	MV*	WK						
	Block Lengths								
			83	113	196	226	422	452	874
<b>Brazil</b>	18.30***	30.77***	- 4.52***	- 4.00***	- 3.24***	- 3.06***	- 2.39***	- 2.33***	- 1.80***
<b>Russia</b>	11.30***	28.80***	- 3.34***	- 3.00***	- 2.51***	- 2.39***	- 1.95***	- 1.91***	- 1.55***
<b>India</b>	14.68***	26.63***	- 4.36***	- 3.83***	- 3.04***	- 2.86***	- 2.21***	- 2.15***	- 1.68***
<b>China</b>	17.09***	29.19***	- 4.68***	- 4.08***	- 3.19***	- 2.99***	- 2.28***	- 2.21***	- 1.70***
<b>S-Africa</b>	17.76***	29.20***	- 4.51***	- 3.97***	- 3.18***	- 3.00***	- 2.34***	- 2.27***	- 1.74***
<b>Pakistan</b>	15.90***	27.31***	- 4.76***	- 4.22***	- 3.40***	- 3.21***	- 2.47***	- 2.40***	- 1.79***

MV: Chow- Denning test; MV\*: Kim test; WK: Whang –Kim test \* means significant at 1% level. The 1% critical value for the Chow-Denning test is 3.022. Following Whang and Kim (2003), block lengths are chosen from an equally spaced grid in the interval  $[2.5T^{0.3}, 3.5T^{0.6}]$ .

## 5 Conclusion

This study focuses upon the existence of random walk in BRICS-P countries through variance ratio test methods contain individual tests; Lo- MacKinlay, Wrights (2000)'s ranks and sign tests and joint ratio tests the Chow- Denning tests, Whang and Kim (2003) and Kim (2006). The joint VR tests independence resting on an asymptotic approximation for statistics sampling distribution is the reason behind applying along with individual variance ratio test.

The results denote that the indices of Brazil, Russia, India, China, South Africa and Pakistan are not showing any evidence of random walk under the time spanning from (January 2000 to March 2017). Other than Brazil, hence these findings are consistent with the recent study of Chong (2010) where Brazil found most efficient in BRIC sample and inconsistent with the evidence of weak form efficiency of Mobarek, (2014). Findings show intertemporal predictability of stock markets of the sample emerging countries, which majorly relates with investors astute. Efficient market hypothesis addresses the investors risk premium for future stock prices which they want to capture risk-adjusted abnormal returns by relying on the information of past stock prices.

The presence of intertemporal predictability in BRICS-P countries rejects the random walk hypothesis and this can pave the way for two probable factors. Firstly, Abraham (2002) relates this situation with an effect of thinly traded stock of listed and small firms in the market. Secondly, study of Hahn (2000) enlightens this situation due to the widespread information asymmetry's effects among market participants.

Finally, this study strengthens the robustness of new VR tests (Wright's & Whang & Kim) sub-sampling techniques for evaluation of Random walk hypothesis besides the previous Lo-MacKinlay & Chow-Denning VR tests for testing Random walk hypothesis.

## References

- Appiah-Kusi, J., & Menyah, K. (2003). Return predictability in African stock markets. *Review of financial economics*, 12(3), 247-270.
- Bachelier, L. (1900). *Theorie de la speculation*: Gauthier-Villars.
- Black, F. (1971). Implications of the random walk hypothesis for portfolio management. *Financial Analysts Journal*, 27(2), 16-22.
- Borges, M. R. (2010). The efficient market hypothesis in European stock markets. *The European Journal of Finance*, 16(7), 711-726.
- Butler, K. C., & Malaikah, S. J. (1992). Efficiency and inefficiency in thinly traded stock markets: Kuwait and Saudi Arabia. *Journal of Banking & Finance*, 16(1), 197-210.
- Chakravarty, S., Sarkar, A., & Wu, L. (1998). Information asymmetry, market segmentation and the pricing of cross-listed shares: Theory and evidence from Chinese A and B shares. *Journal of International Financial markets, institutions and money*, 8(3), 325-356.

- Charles, A., & Darné, O. (2009). The random walk hypothesis for Chinese stock markets: Evidence from variance ratio tests. *Economic Systems*, 33(2), 117-126.
- Chong, T. T.-L., Cheng, S. H.-S., & Wong, E. N.-Y. (2010). A comparison of the stock market efficiency of the BRIC countries. *Technology and Investment*, 1(04), 235.
- Chow, K. V., & Denning, K. C. (1993). A simple multiple variance ratio tests. *Journal of Econometrics*, 58(3), 385-401.
- Cooray, A., & Wickremasinghe, G. (2007). The efficiency of emerging stock markets: Empirical evidence from the South Asian region. *The Journal of Developing Areas*, 41(1), 171-183.
- Fama, E. F. (1965). The behaviour of stock-market prices. *The Journal of Business*, 38(1), 34-105.
- Groenewold, N., Tang, S. H. K., & Wu, Y. (2003). The efficiency of the Chinese stock market and the role of the banks. *Journal of Asian Economics*, 14(4), 593-609.
- Hall, S., Urga, G., & Zalewska-Mitura, A. (1998). *Testing for Evolving Stock Market Efficiency: With an Application to Russian Stock Prices*: London Business School, Centre for Economic Forecasting.
- Jarrett, J. E., & Kyper, E. (2006). Capital market efficiency and the predictability of daily returns. *Applied Economics*, 38(6), 631-636.
- Kim, J. H. (2006). Wild bootstrapping variance ratio tests. *Economics Letters*, 92(1), 38-43.
- Laurence, M. M., Cai, F., & Qian, S. (1997). Weak-form efficiency and causality tests in Chinese stock markets.
- Lim, K.-P., Habibullah, M. S., & Hinich, M. J. (2009). The weak-form efficiency of Chinese stock markets: thin trading, nonlinearity and episodic serial dependencies. *Journal of Emerging Market Finance*, 8(2), 133-163.
- Lo, A. W., & MacKinlay, A. C. (1988). Stock market prices do not follow random walks: Evidence from a simple specification test. *The review of financial studies*, 1(1), 41-66.
- Lo, A. W., & MacKinlay, A. C. (1989). The size and power of the variance ratio test in finite samples: A Monte Carlo investigation. *Journal of Econometrics*, 40(2), 203-238.
- Malkiel, B. G., & Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of finance*, 25(2), 383-417.
- Mobarek, A., & Fiorante, A. (2014). The prospects of BRIC countries: Testing weak-form market efficiency. *Research in International Business and Finance*, 30, 217-232.
- Nalın, H. T., & Güler, S. (2015). Testing The Random Walk Hypothesis: An Application in the BRIC Countries and Turkey. *Romanian Economic Journal*, 18(55).
- Narayan\*, P. K., & Smyth, R. (2004). Is South Korea's stock market efficient? *Applied Economics Letters*, 11(11), 707-710.
- Samuelson, P. A. (1965). Proof that properly anticipated prices fluctuate randomly. *IMR; Industrial Management Review (pre-1986)*, 6(2), 41.
- Summers, L. H. (1986). Does the stock market rationally reflect fundamental values? *The Journal of finance*, 41(3), 591-601.

Whang, Y. J., & Kim, J. (2003). A multiple variance ratio tests using subsampling. *Economics Letters*, 79(2), 225-230.

Worthington, A. C., & Higgs, H. (2004). Random walks and market efficiency in European equity markets. *Global Journal of Finance and Economics*, 1(1), 59-78.

Wright, J. H. (2000). Alternative variance-ratio tests using ranks and signs. *Journal of Business & Economic Statistics*, 18(1), 1-9.

Wu, Y. (1996). Are real exchange rates nonstationary? Evidence from a panel-data test. *Journal of Money, Credit and Banking*, 28(1), 54-63.